

## 5. Integer and Floating Point Arithmetic ENEE 140

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<http://ter.ps/enee140>

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### Today's Lecture

- Where we've been
  - Using variables and constants
  - Variable assignment and operators
  - Iterating (`while`, `for`) and branching (`if`)
  - `printf()` and `scanf()`
  - Functions
  - Basics of data types
- Where we're going today
  - Unsigned data type
  - The `%` operator
  - Prefix and postfix increment operators
  - Assignment operators (`+=`)
  - Integer & floating point arithmetic
  - Overflow and underflow
- Where we're going next
  - Data types and type conversion

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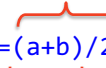
### Review: Integer vs. Floating-Point Arithmetic

- What is the value of c?

```
float a = 1;
```

```
float b = 2;
```

```
int c=(a+b)/2+0.5;
```







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### Review: Integer vs. Floating-Point Arithmetic – cont'd

- What is the value of g?

```
int a = 1;
```

```
int b = 2;
```

```
float c = 1;
```

```
float d = 2;
```

```
float e = c/d;
```

```
int f = a/d;
```

```
int g;
```

```
if (e > (float)f) {
```

```
    g = 1;
```

```
} else {
```

```
    g = 0;
```

```
}
```




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## Prefix and Postfix Increment Operators

- We've seen: increment and decrement operators in C
  - `a++;` same as `a = a + 1;`
  - `++a;` same as `a = a + 1;`
  - `a--;` same as `a = a - 1;`
  - `--a;` same as `a = a - 1;`
- We've also seen: value of assignment expressions
  - `c = b = a = 0;` a, b and c become 0
- Both `a++` and `++a` increment a, but they return different values
  - `b = a++;` postfix increment: a becomes 1, b becomes 0
  - `b = ++a;` prefix increment: a becomes 1, b becomes 1
  - Same for `a--` and `--a`

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## Assignment Operators

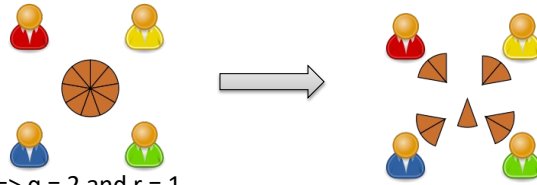
- We've seen
  - `a++;` increment a by 1
  - `a = a + 10;` increment a by 10
- Prefix and postfix operators
  - `a = 1;`
  - `b = a++;` a becomes 2, b becomes 1
  - `b = ++a;` a becomes 3, b becomes 3
- Other assignment operators
  - `a += 10;` increment a by 10
  - Same for `--`, `*=`, `/=`, `%=`

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## Integer (Euclidean) Division – In Math

- Dividing two integers produces a **quotient (q)** and a **remainder (r)**
  - The quotient and the remainder always exist and are unique
  - Example: dividing a pie with 9 slices among 4 people (source: Wikipedia)



–  $9 / 4 \Rightarrow q = 2$  and  $r = 1$

- Mathematical definition:
  - Given integers  $A$  and  $B$ , with  $B \neq 0$ : there exist **unique** integers  $r, q$  such that:
    - $A = B \cdot q + r$  and
    - $0 \leq r < B$

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## Integer (Euclidean) Division - Examples

- What are the remainders and quotients when dividing:

▪ 8 by 4

▪ 4 by 8

▪ 10 by 10

- What is the remainder when dividing:

▪  $(2^n - 1)$  by 2

▪  $2 \cdot n$  by  $n$

▪  $(2^n - 1)$  by  $2^n$

(assume that  $n$  is a positive integer)

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## Computer Arithmetic – Operations

- We've seen
  - + - / \* arithmetic operators
  - Work for both integer and floating-point variables
  - Integer division truncates toward 0 (i.e. the fractional part is discarded)
- The modulus operator %
  - Works only for integers
  - Produces the remainder from integer division

```
int a = 5 / 3;    value of a is 
```

```
int b = 5 % 3;   value of b is 
```

- The values of `a % n` range between

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## Order of Evaluation

- Operator precedence (complete rules in K&R Table 2.1)
  1. ! ++ -- (unary operators)
  2. \* / %
  3. + -
  4. < <= > >=
  5. == !=
  6. &&
  7. ||
  8. =
- Rule of thumb:
  - Division and multiplication come before addition and subtraction
  - Put parentheses around everything else

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## Unsigned Data Types

- We've seen
 

```
int a = -1;
long b = -1;
```
- Unsigned data types are always positive
 

```
unsigned a = 1;
unsigned long b = 1;
```
- Unsigned literals
 

<code>1U</code>	1 as unsigned constant
<code>1LU</code>	1 as unsigned long constant

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## Limits for Computer Integers

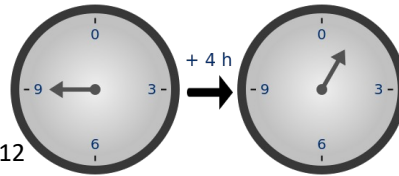
- Limits for unsigned integers (`unsigned`, not `unsigned long`)
  - `UINT_MIN` = 0
  - `UINT_MAX` =  $2^w - 1$
- Limits for signed integers (`int`, not `long int`)
  - `INT_MIN` =  $-2^{w-1}$
  - `INT_MAX` =  $2^{w-1} - 1$
  - Note: the textbook **incorrectly** implies that `INT_MIN == -INT_MAX` (section B11)
- `w` is machine dependent
  - `w` = 32 on the GRACE machines
  - `UINT_MAX`, `INT_MIN` and `INT_MAX` are defined as constants in `limits.h`

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## Integer Overflow

- What happens when you add 1 to `UINT_MAX`?
  - The mathematical value  $(2^w - 1) + 1 = 2^w$  cannot be stored in an **unsigned** variable
  - The result of the operation is 0
- Intuition: unsigned numbers **wrap around**
  - Think of a 12h clock
  - 11 o'clock + 1h = 0
  - We count time **modulo 12h**
  - This means that the time displayed is the **remainder** from a division by 12



Source: Wikipedia

- **Unsigned operations are done modulo  $2^w$**

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## Unsigned Integer Addition

- Mathematical addition: **infinite** precision
  - $s$   $= u + v$
- **unsigned** addition: operations done **modulo  $2^w$** 
  - $s$   $= (u + v) \bmod 2^w$
  - Example: `UINT_MAX + 1`  $= 2^w \bmod 2^w = 0$  (**overflow**)
- Multiplication can overflow in similar manner
  - Same for addition and multiplication of signed integers

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## Properties of Signed Integers – Examples

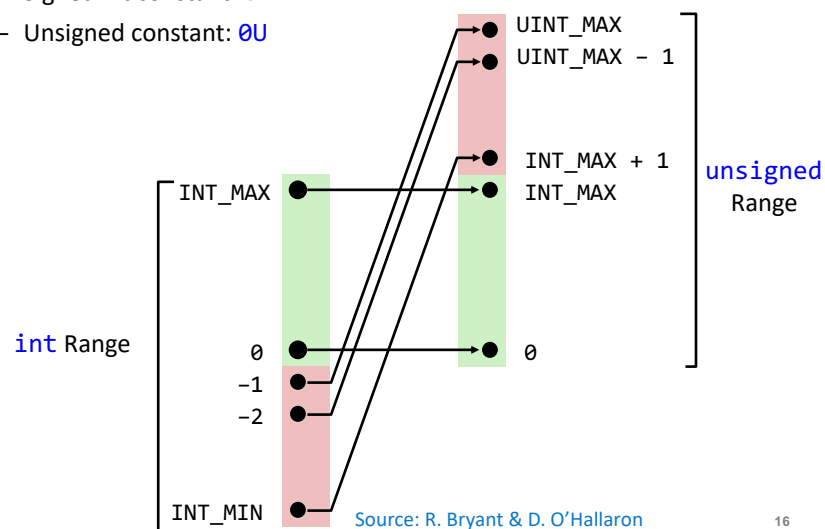
- You can represent more negative than positive numbers
  - Positive range:  $1 \dots (2^{w-1} - 1)$
  - Negative range:  $-1 \dots -2^{w-1}$
- Signed integers can overflow as well
  - $\text{INT\_MAX} + 1 = \text{INT\_MIN}$ 
    - Adding 2 positive numbers may produce a negative number!
  - $\text{INT\_MIN} - 1 = \text{INT\_MAX}$ 
    - Adding 2 negative numbers may produce a positive number!
  - $\text{INT\_MIN} = -\text{INT\_MIN}$ 
    - $\text{INT\_MIN}$  is its own inverse

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## Conversion Between Signed and Unsigned

- Type conversion `int`  $\rightarrow$  `unsigned` visualized
  - Signed int constant: `0`
  - Unsigned constant: `0U`



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## Mathematical Properties of Integer Arithmetic

- Closed under addition and multiplication
  - Result of signed/unsigned operation is also a signed/unsigned integer
- Commutative
- Associative
- 0 is additive identity; 1 is multiplicative identity
- Multiplication distributes over addition
  - $a * (b + c) = a*b + a*c$
- **Does not obey the ordering properties** of math integers

$$\begin{array}{ll} u > 0 & \not\Rightarrow u + v > v \\ u > 0, v > 0 & \not\Rightarrow u \cdot v > 0 \end{array}$$

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## Properties of Floating Point Numbers

- As many negative as positive numbers
- Special values (constants for some of these defined in `float.h`)
  - Max floating point number  $\Rightarrow$  operations may **overflow**
  - Min floating point  $> 0$   $\Rightarrow$  operations may **underflow**
  - Smallest  $\epsilon$  such that  $1.0 + \epsilon \neq 1.0 \Rightarrow$  operation results may be **rounded**
  - +Inf, -Inf, NaN (not a number)

- **Avoid testing the equality** of values resulting from floating point operations

```
if (FLT_MAX == (FLT_MAX+1)) {...}    condition is true
if (cos(M_PI / 2) != 0.0) {...}     condition is true
```

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## Mathematical Properties of Floating Point Arithmetic

- Closed under addition and multiplication
  - But may generate infinity or NaN
- Commutative
- **Not associative**
  - $(a + b) + c \neq a + (b + c)$
  - $(a * b) * c \neq a * (b * c)$
  - Possibility of overflow, inexactness of rounding
- Multiplication **does not distribute** over addition
  - $a * (b + c) \neq a * b + a * c$
  - Possibility of overflow, inexactness of rounding
- Monotonicity
  - $a \geq b \quad \Rightarrow a+c \geq b+c$
  - $a \geq b \ \& \ c \geq 0 \quad \Rightarrow a * c \geq b * c$
  - **Exceptions:**  $\pm\text{Inf}$  and NaN

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## Review of Lecture

- What did we learn?

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## Next Steps

- Next lecture
  - Data types and type conversion
- Assignments for this week
  - Read **K&R Chapters 2.2, 2.9, 3.3, 6.1, B5, B6**
    - Note: some of these chapters refer to strings (e.g. `char s[]`), which we'll cover later
    - For now, think of `s[i]` as a character variable
  - Weekly challenge: **dec2bin.c**
  - Homework: **lab05.pdf** (on <http://ter.ps/enee140>), due on Friday at 11:59 pm
  - No quiz next week

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